



**MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 2**

Thursday 4 May 2000 (morning)

2 hours

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all five questions from Section A and one question from Section B.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures as appropriate.
- Write the make and model of your calculator on the front cover of your answer booklets *e.g.* Casio *fx-7400G*, Sharp EL-9400, Texas Instruments TI-80.

You are advised to start each new question on a new page. A correct answer with **no** indication of the method used will usually receive **no** marks. You are therefore advised to show your working. (If graphs from a graphic display calculator are being used to find solutions, you should sketch these graphs as part of your answer.)

SECTION A

Answer all **five** questions from this section.

1. [Maximum mark: 15]

The cost of sending documents from Australia to the United States of America using *Immediate Courier* is recorded in the table below.

Weight in kilograms (<i>W</i>)	Cost in Australian dollars (AUD) (<i>C</i>)
$0.0 < \text{Weight} \leq 0.5$	20
$0.5 < \text{Weight} \leq 1.0$	30
$1.0 < \text{Weight} \leq 1.5$	40
$1.5 < \text{Weight} \leq 2.0$	50
$2.0 < \text{Weight} \leq 2.5$	56
$2.5 < \text{Weight} \leq 3.0$	62
$3.0 < \text{Weight} \leq 3.5$	68
$3.5 < \text{Weight} \leq 4.0$	74

- (a) On graph paper, using a scale of 2 cm to represent 1 kg on the horizontal axis and 2 cm to represent 10 AUD on the vertical axis, draw and label the graph, representing this information.

[5 marks]

(This question continues on the next page)

(Question 1 continued)

The cost of sending documents from Australia to the United States of America using *Speedy Courier* is given by the formula

$$C = 34 + 8W, \quad \text{where } C \text{ is the cost in Australian dollars,}$$
$$W \text{ is the weight in kilograms.}$$

(b) (i) Copy and complete the table below.

W	0.5	1	2	3	4
C		42			

[2 marks]

(ii) On the same axes as in part (a), draw the graph representing *Speedy Courier* costs.

[2 marks]

(c) Use your graph to find

(i) the weight for which the cost is the same for both companies ;

[1 mark]

(ii) the difference in cost to send a parcel weighing 3.5 kg .

[2 marks]

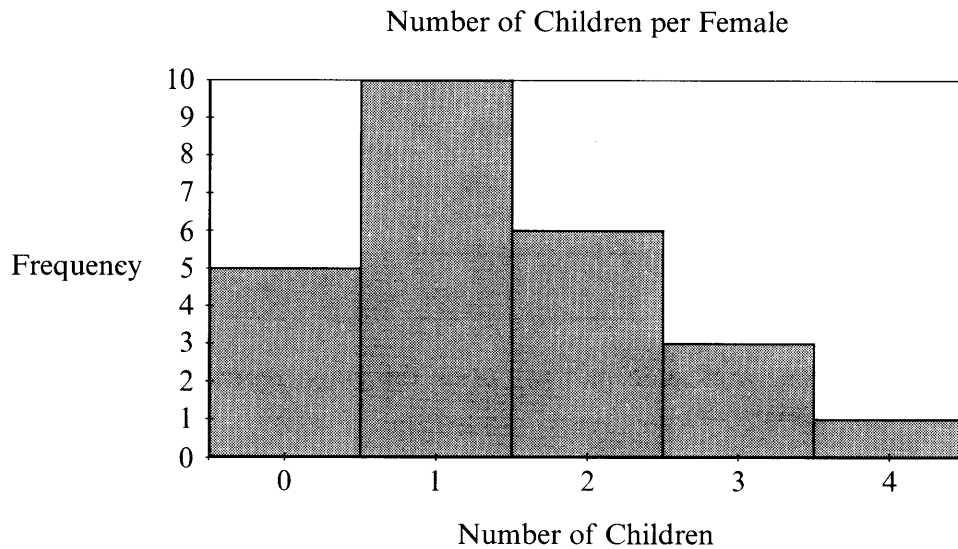
The maximum weight for any package is 4 kg. Documents weighing more than 4 kg have to be put into separate packages.

(d) Kim spent 106 AUD to send 2 packages of documents with a combined weight of 5.5 kgs. Given that he used both courier services, calculate the weight of documents he sent through each one.

[3 marks]

2. [Maximum mark: 15]

A group of 25 females were asked how many children they each had. The results are shown in the histogram below.



- (a) Show that the mean number of children per female is 1.4 . [2 marks]
- (b) Show clearly that the standard deviation for this data is approximately 1.06 . [3 marks]
- (c) Another group of 25 females was surveyed and it was found that the mean number of children per female was 2.4 and the standard deviation was 2 . Use the results from parts (a) and (b) to describe the differences between the number of children the two groups of females have. [2 marks]
- (d) A female is selected at random from the first group. What is the probability that she has more than two children? [2 marks]
- (e) Two females are selected at random from the first group. What is the probability that
 - (i) both females have more than two children? [2 marks]
 - (ii) only one of the females has more than two children? [3 marks]
 - (iii) the second female selected has two children given that the first female selected had no children? [1 mark]

3. [Maximum mark: 15]

- (i) A group of 30 children are surveyed to find out which of the three sports cricket (*C*), basketball (*B*) or volleyball (*V*) they play. The results are as follows :

3 children do not play any of these sports
2 children play all three sports
6 play volleyball and basketball
3 play cricket and basketball
6 play cricket and volleyball
16 play basketball
12 play volleyball.

- (a) Draw a Venn diagram to illustrate the relationship between the three sports played. [1 mark]
- (b) On your Venn diagram indicate the number of children that belong to each region. [3 marks]
- (c) How many children play only cricket? [2 marks]

- (ii) Let the propositions *p*, *q* and *r* be defined as :

p : Matthew arrives home before six o'clock
q : Matthew cooks dinner
r : Jill washes the dishes

- (a) (i) Express the following statement in logical form.

If Matthew arrives home before six o'clock then he will cook dinner. [1 mark]

- (ii) Write the following logic statement in words.

$$\neg q \Rightarrow \neg r \quad [1 \text{ mark}]$$

(This question continues on the next page)

(Question 3 continued)

(b) (i) Copy and complete the truth table below.

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$\neg r$	$(p \Rightarrow q) \wedge (q \Rightarrow r) \wedge \neg r$	$\neg p$	$[(p \Rightarrow q) \wedge (q \Rightarrow r) \wedge \neg r] \Rightarrow \neg p$
T	T	T						T
T	T	F						T
T	F	T						T
T	F	F						T
F	T	T						T
F	T	F						T
F	F	T						T
F	F	F						T

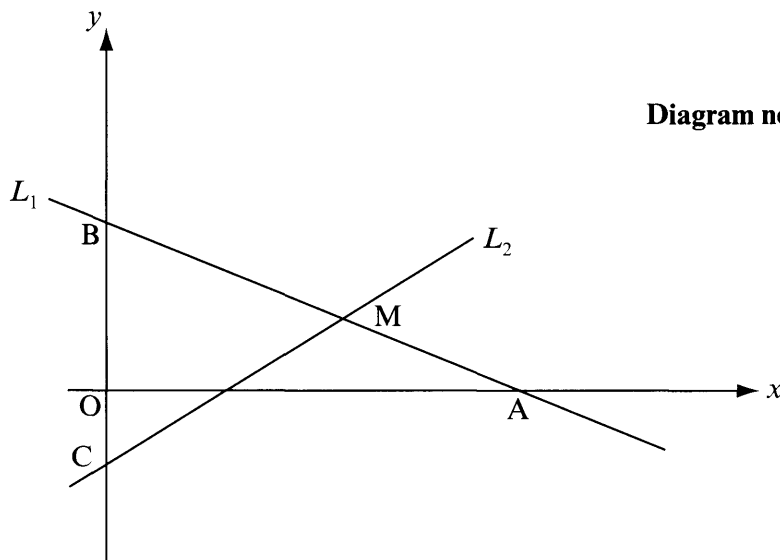
[5 marks]

(ii) Explain the significance of the truth table above.

[2 marks]

4. [Maximum mark: 15]

The line L_1 shown on the set of axes below has equation $3x + 4y = 24$. L_1 cuts the x -axis at A and cuts the y -axis at B.



(a) Write down the coordinates of A and B. [2 marks]

M is the mid point of the line segment [AB].

(b) Write down the coordinates of M. [2 marks]

The line L_2 passes through the point M and the point C (0, -2).

(c) Write down the equation of L_2 . [2 marks]

(d) Find the length of

(i) MC; [2 marks]

(ii) AC. [2 marks]

(e) The length of AM is 5. Find

(i) the size of angle CMA; [3 marks]

(ii) the area of the triangle with vertices C, M and A. [2 marks]

5. [Maximum mark: 10]

Miranti deposits \$1000 into an investment account that pays 5% interest per annum.

- (a) What will be the value of the investment after 5 years if the interest is reinvested? [2 marks]
- (b) How many years would it take Miranti's investment of \$1000 to double in value? [4 marks]

At the beginning of each year Brenda deposits \$1000 into an investment account that pays 5% interest per annum. Interest is calculated annually and reinvested.

- (c) How much would be in Brenda's account after 5 years? [4 marks]

SECTION B

Answer one question from this section.

Matrices and Graph Theory

6. [Maximum mark: 30]

- (i) A secondary school is made up of eight separate buildings named A, B, C, D, E, F, G and H. The buildings are connected by pathways as shown on the diagram. The distances, in metres, between buildings are shown on the diagram.

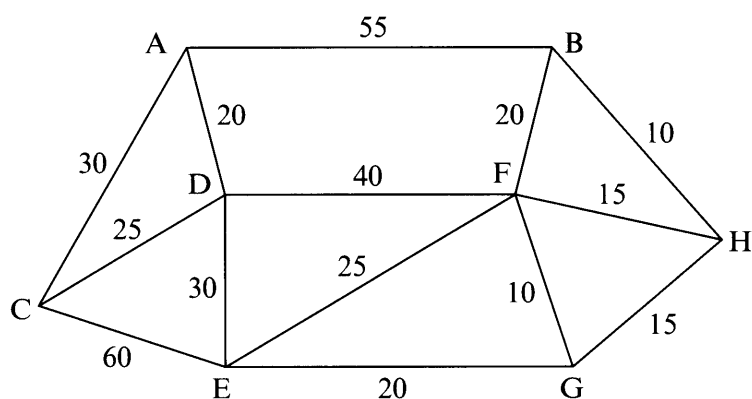


Diagram not drawn to scale

- (a) Every afternoon a cleaner collects the rubbish from each building. He starts and finishes at H and he wishes to walk the minimum distance possible.
- (i) Show the path he takes. [3 marks]
- (ii) What is the minimum distance he has to walk? [2 marks]
- (b) The principal of the school wishes to construct some covered walkways so that staff and students can travel between buildings on rainy days without getting wet. To minimise the cost the principal wants to construct the minimum length of total covered pathway.
- (i) Draw a diagram to show which buildings would be connected with the covered pathway. [3 marks]
- (ii) What is the minimum length of covered pathway required? [2 marks]

(This question continues on the next page)

(Question 6 continued)

- (ii) A , B , C and D are towns connected by canals. Some canals are only wide enough for boats to travel in one direction; other canals are wide enough for boats to travel in both directions. The following matrix M shows the number of ways of travelling from one town to another by canal.

		To				
		A	B	C	D	
From	A	(0	2	0	1
	B		1	0	0	0
	C		0	1	0	0
	D		0	1	1	0
)				

- (a) Is it possible to go from town C to town D without going through another town? [1 mark]
- (b) How many canals can you use to go from town A to town B without going through another town? [1 mark]
- (c) Given that

$$M^2 = \begin{pmatrix} a & 1 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 1 & 0 & 0 & b \\ 1 & 1 & 0 & 0 \end{pmatrix}$$

calculate the value of

- (i) a ; [2 marks]
- (ii) b . [2 marks]
- (d) Is it possible to go from town D to town B passing through one other town? [1 mark]
- (e) Draw the directed graph which is represented by the incidence matrix below.

		To				
		A	B	C	D	
From	A	(0	2	1	0
	B		0	0	0	0
	C		1	1	1	1
	D		1	0	0	0
)				

[4 marks]

(This question continues on the next page)

(Question 6 continued)

(iii) A small town has 10 000 people who are able to work. This year 7500 of these are employed and 2500 are unemployed. Under the present economic conditions it is known that next year the total working population will remain the same, 10% of those presently employed will become unemployed and 15% of those presently unemployed will become employed.

(a) (i) Copy and complete the transition matrix T below to represent the information given.

		This year	
		Employed	Unemployed
Next Year	Employed	$\begin{pmatrix} 0.9 & \end{pmatrix}$	
	Unemployed		

[3 marks]

(ii) Hence calculate the number of people who will be unemployed next year.

[2 marks]

(b) Given that

$$T^2 = \begin{pmatrix} 0.825 & 0.2625 \\ 0.175 & 0.7375 \end{pmatrix}$$

and

$$T^2 \begin{pmatrix} 7500 \\ 2500 \end{pmatrix} = \begin{pmatrix} m \\ n \end{pmatrix}$$

(i) find the value of m , to four significant figures;

[2 marks]

(ii) explain what this value of m represents.

[2 marks]

Further Statistics and Probability

7. [Maximum mark: 30]

(i) A machine produces bolts whose diameters are required to be a certain size. When the machine is operating properly, the diameters are normally distributed with mean 50 mm and standard deviation 4 mm.

(a) What is the probability that a bolt selected at random has a diameter

(i) less than 40 mm? [3 marks]

(ii) between 40 mm and 45 mm? [3 marks]

(iii) between 45 mm and 58 mm? [3 marks]

Constant use puts the machine out of adjustment. Every week 100 bolts are tested to ensure that the machine is operating to its required level of performance. The results of one test are summarised in the table below.

Bolt diameter	Observed frequency	Expected frequency
$D < 46$	11	16
$46 \leq D < 50$	29	34
$50 \leq D < 54$	33	34
$54 \leq D < 58$	18	14
$58 \leq D$	9	2
Totals	100	100

The χ^2 test for goodness of fit is used to determine whether this sample fits the required distribution.

(b) (i) Write the null (H_0) and alternative (H_1) hypotheses for this test. [2 marks]

(ii) Calculate the chi-squared statistic. [4 marks]

(iii) Determine, to a 5% significance level, if the machine needs to be adjusted. Justify your answer. [3 marks]

(This question continues on the next page)

(Question 7 continued)

- (ii) The *Type Fast* secretarial training agency has a new computer software spreadsheet package. The agency investigates the number of hours it takes people of varying ages to reach a level of proficiency using this package. Fifteen individuals are tested and the results are summarised in the table below.

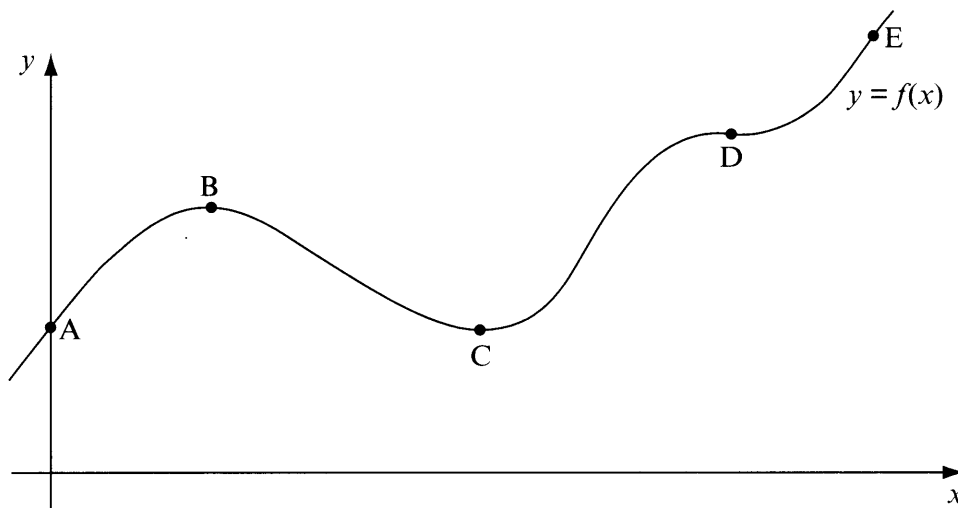
Age (x)	32	40	21	45	24	19	17	21	27	54	33	37	23	45	18
Time (in hours) (y)	10	12	8	15	7	8	6	9	11	16	t	13	9	17	5

- (a) (i) Given that $S_y = 3.5$ and $S_{xy} = 36.7$, calculate the product-moment correlation coefficient r for this data. [4 marks]
- (ii) What does the value of the correlation coefficient suggest about the relationship between the two variables? [1 mark]
- (b) Given that the mean time taken was 10.6 hours, write the equation of the regression line for y on x in the form $y = ax + b$. [3 marks]
- (c) Use your equation for the regression line to predict
- (i) the time that it would take a 33 year old person to reach proficiency, giving your answer correct to the nearest hour; [2 marks]
- (ii) the age of a person who would take 8 hours to reach proficiency, giving your answer correct to the nearest year. [2 marks]

Introductory Differential Calculus

8. [Maximum mark: 30]

(i) A, B, C, D and E are points on the curve $y=f(x)$ shown in the diagram below.



(a) For each of the following points, state whether it is a maximum point, a minimum point, or a point of inflexion.

(i) B ; [1 mark]

(ii) D . [1 mark]

(b) Describe the gradient of the curve in passing from the point B, through point C to point D. [3 marks]

(c) D has coordinates $(a, f(a))$, and the x -coordinate at E is $a + 4$. Write an expression for the gradient of the line segment [DE]. [3 marks]

(ii) The distance in kilometres (k) of a cyclist from his home after t hours is given by

$$k = t^3 - 3t^2 + 3t + 4$$

(a) How far is the cyclist from his home after 4 hours? [2 marks]

(b) (i) Find $\frac{dk}{dt}$. [2 marks]

(ii) Hence or otherwise find the speed of the cyclist at $t = 3$. [2 marks]

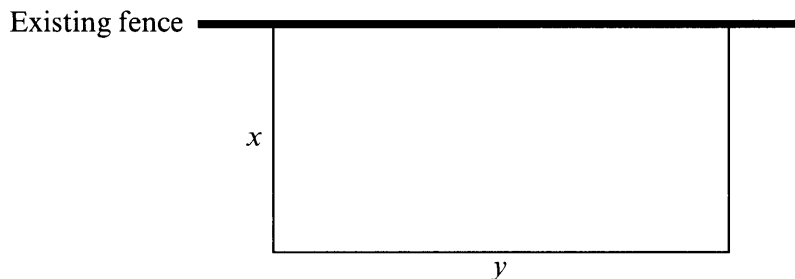
(iii) Find the value of t when the cyclist is momentarily stopped. [3 marks]

(iv) How far is the cyclist from his home at this point? [2 marks]

(This question continues on the next page)

(Question 8 continued)

- (iii) A farmer wishes to enclose a rectangular field using an existing fence for one of the four sides.



- (a) Write an expression in terms of x and y that shows the total length of the new fence. [1 mark]
- (b) The farmer has enough materials for 2500 metres of new fence. Show that

$$y = 2500 - 2x \quad \text{[1 mark]}$$

- (c) $A(x)$ represents the area of the field in terms of x .

- (i) Show that

$$A(x) = 2500x - 2x^2 \quad \text{[2 marks]}$$

- (ii) Find $A'(x)$. [1 mark]

- (iii) Hence or otherwise find the value of x that produces the maximum area of the field. [3 marks]

- (iv) Find the maximum area of the field. [3 marks]

